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**Citation for published version:**

Hopkins, E & Kornienko, T 2009 'Status, Affluence, and Inequality: Rank-Based Comparisons in Games of Status' Levine's Bibliography, UCLA Department of Economics, pp. 1-32.  
<<http://ideas.repec.org/p/cla/levrem/12224700000001442.html>>

**Link:**

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**Document Version:**

Early version, also known as pre-print

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# Status, Affluence, and Inequality: Rank-Based Comparisons in Games of Status\*

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January, 2009

## Abstract

We examine the effects of changes in the income distribution in an economy where agents' utility depends both on consumption and on their rank in the distribution of conspicuous consumption. We introduce a new methodology to compare the behavior of agents that occupy the same rank in the two different income distributions but typically have different levels of incomes. Here, an increase in incomes of the least endowed improves their welfare, yet it also increases social competition, making those in the middle to be worse off - even if they have higher incomes as well. As social competition can be lowered by spreading agents apart in income space, we find that an increase in incomes for all, augmented by (weakly) increased income dispersion, constitutes a sufficient condition for Pareto-improvement. We also show that one can have an increase both in income and relative position but still be worse off.

*Journal of Economic Literature* classification numbers C72, D11, D31, D62

*Keywords:* Status, relative standing, conspicuous consumption, consumption externalities, income inequality, social competitiveness, first price auctions, dispersive orderings.

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\*We thank John Duffy, Nick Hanley, R. Vijay Krishna, Benny Moldovanu, József Sákovics and Jonathan Thomas for comments. Errors remain our own. This paper previously circulated as "Methods of Social Comparison in Games of Status". Ed Hopkins acknowledges financial support from the Economic and Social Research Council, award reference RES-000-27-0065 and a Leverhulme Trust Study Abroad Fellowship. Tatiana Kornienko acknowledges financial support from the Faculty of Management of the University of Stirling.

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# 1 Introduction

The standard argument in support of economic growth rests on the assumption that an increase in real income of every individual brings an increase in happiness for all. However, Easterlin (1974, 1995) pointed out that data on happiness across time and countries did not support this line of thought. Concerns with relative position are a likely culprit, in that happiness increases significantly in cross-section even if average happiness does not rise strongly in response to increases in average income. This has led to an increasing acceptance amongst economists that people may care about their relative position as well as the absolute level of their consumption. For example, a survey of empirical research on happiness goes as far as to conclude that in determining happiness “It is not the absolute level of income that matters most but rather one’s position relative to other individuals” (Frey and Stutzer, 2002, p411). The importance of relative comparisons finds further support in more recent research (Brown et al. (2008); Clark et al. (2008); Luttmer (2005)).

This raises two important questions. First, if relative concerns can weaken the connection between economic growth and greater happiness, are there any conditions under which all people in a society can be made happier by a rise in incomes, given the presence of relative concerns? Second, if people do have relative concerns, it seems a natural conclusion that greater inequality would lower the happiness of the middle classes as distance between them and the rich becomes larger (Frank (1999)). This would seem to give a new justification for policies to reduce inequality. But what exactly is the relationship between equality and happiness when there are relative concerns?

In our current work, we analyze a model derived from that of Frank (1985), where individuals decide how to divide their income between consumption of a normal good and a positional good. For example, one might care about the characteristics of one’s car, but also about how it compares to those of one’s neighbors. The choice of the positional good is therefore strategic, in that consumption choices of my neighbors affect my payoffs, and my choice affects theirs. The symmetric Nash equilibrium of the resulting game will be Pareto inefficient in that all will spend more on the positional good than is privately optimal, but will result in no net change in relative position. We find that greater equality provides greater incentives to spend on the visible positional good as it becomes easier to surpass one’s neighbors. An increase in equality that raises incomes at the lower end of the income distribution will make the poor better off. But as an increase in equality increases the degree of social competition, the middle class will be worse off - even if they have higher incomes as well. We find that even in the presence of this social competition it is possible for economic growth to make all better off: it is sufficient to raise incomes provided that at the same time there is no increase in income density, or equivalently, no decrease in income dispersion.

In such a model changes in the income distribution can affect agents through three channels. One’s own income, one’s relative position or rank in the distribution, and the shape of the distribution all matter. Of necessity, any analysis must hold at least of one

of these constant. In our earlier work (Hopkins and Kornienko (2004)), we considered changes in the distribution of income that left some people's incomes unchanged. (For example, imagine a change in taxes on earned income that does not affect incomes of those who do not work.) We then analysed the effect of the change in the distribution of income on equilibrium behavior and utility at each income level. We found that a reduction of inequality of this type would lead to a fall in utility at low income levels.

In this paper, we develop a new set of techniques created specifically for problems of social comparison. We re-examine games of status where agents care about their relative position in terms of their rank in expenditure on a visible positional good, and analyze consumer choice as a function of *rank* rather than *income*. This new methodology allows us to compare the equilibrium behavior and well-being of every individual for a pair of continuous income distributions, each having support on an interval, but these intervals may have all, some, or no points in common.

Here, we find, as we did before, that an increase in equality increases the degree of social competition. The closer individuals are together, the easier it is to overtake others in status, thus giving a greater incentive to indulge in conspicuous consumption. This means that the overall effect of redistribution on the poor is ambiguous: they have greater income, but more of it may be spent on wasteful consumption. Typically, with their income increased, the poor are better off in a more equal society, yet the lower middle class are worse off - even though they may also have higher income. This is because the increased affluence of those at the bottom results in their higher expenditure on visible positional goods, forcing everyone further up the social ladder to increase their spending as well in order to "keep up" - and for most, this increased expenditure on positional goods comes at the expense of their expenditure of non-positional goods, leading to a decline in post-redistribution welfare. The effect on those with average or greater income is definitely negative. This is in contrast to the effect of a reduction in inequality in Hopkins and Kornienko (2004), where, for a given level of income, the effect on the rich was ambiguous, and on the poor was definitely negative.

We hope these seemingly contrasting results may help to explain why it has been difficult to establish empirically whether greater equality does in fact lead to greater happiness. Clark (2003), using British panel data, finds a positive relationship between inequality and self-reported happiness while Senik (2004) finds that inequality has no statistical influence on life satisfaction in post-reform Russia. In contrast, Alesina *et al.* (2004) find a negative relationship between inequality and happiness for both Europe and the US. Our results suggest that, even in the presence of relative concerns, whether greater equality does increase utility or happiness may depend quite sensitively on the measure of equality considered and on the method of comparison.

In this paper, we also explore under what conditions an increase in the income of each person makes everyone happier. We find that all agents will be better off when all have an increase in income, and, further, incomes become no less dispersed. That is, to prevent an increase in "social competitiveness" (by which we mean an increase in the density of the income distribution), this increase in affluence should spread agents

apart - or at least not squeeze them together - in income space. That is, growth leads to greater happiness if it raises the incomes of all without increasing equality.

Notice that our techniques allows us to avoid the problems of interpersonal comparisons as we can make ordinal comparisons of utilities for the same individual before and after an income transformation. If the income transformation is rank-preserving, each individual will have the same rank before and after the income transformation, and thus we can apply our rank-indexing technique directly. We find that even if the income transformation is not rank-preserving, one can still carry out ordinal comparisons for the same individual by correcting for ranks by means of a rank transformation function. Non-rank-preserving transformations highlight the importance of the density of the social space, as we provide an example where individuals may have both higher income and higher rank, but because of the increased density of their social space they may be worse off.

Both the model presented here and the model in Hopkins and Kornienko (2004) can also be interpreted as models of labor supply, as they can apply to situations where individuals decide on how to allocate their endowment of productivity (rather than of income) between labor and leisure, and where status or prestige is assigned to the most productive workers. In equilibrium individuals choose a level of labor which is higher than that which would be privately optimal - that is, in terms of labor supply, they overwork. As increased education changes the productivity endowments (making the “social space” to be less dispersed), the increased competition leads to further worker dissatisfaction.<sup>1</sup>

One important aspect of our model is that, following Frank (1985) and Robson (1992), we assume that an individual’s status is determined by her position in the distribution of conspicuous consumption, with higher position meaning higher status. That is, here we assume status is ordinal, and depends on how many others one surpasses. An alternative form would be some form of cardinal status, depending, for example, on the distance between one’s own and others’ consumption. The cardinal case has been analysed by Clark and Oswald (1998) and Friedman and Ostrov (2008) (see also Hopkins (2008) for a brief survey) but not for the case of a heterogeneous population considered here.

Status is necessarily defined relative to some comparison group. Here, this is assumed to be the whole population rather than a smaller group, such as those having similar incomes or those in the local neighborhood. Defining the appropriate comparison group is an important task for empirical work on relative concerns - see, for example, the survey by Clark et al. (2008). However, there are strong arguments that the comparison across the population is the most important. As Wilkinson and Pickett (2006, p1776) put it, “our recognition of our class status is constituted primarily by our recognition of uncontested status differences”. In other words, concentrating our comparisons on those who are near to us in status is only possible after classifying who

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<sup>1</sup>See Schor (1991) and (1998) for a vivid description of both overspending and overworking behavior by modern Americans.

is close and who is not. That is, it requires some recognition of one's global position, rich, poor or somewhere between.

The ordinal specification is, in fact, very sensitive to local conditions. As we show, what drives an individual's behavior is the income density at her rank in the income distribution. The point is that individuals with a concern with rank wish to rise in the rankings. To do so, they are in competition with those around them. They might like to be at the top of the distribution, but what is important for equilibrium behavior is their competition with their neighbors.

Finally, our work adds to a growing literature in economics on the theory of relative concerns. Here, for a general specification of primitives, we look at how macroeconomic aggregates such as economic growth and inequality affects individual decisions and welfare. The earlier literature (including Cole, Mailath and Postlewaite (1992), Corneo and Jeanne (1997, 1999), Cooper, García-Peñalosa and Funk (2001), and many others) looked instead at the effects of relative concerns on economic growth. Dubey and Geanakoplos (2005), Moldovanu, Sela and Shi (2005) and Dhillon and Herzog-Stein (2005) all consider the design of optimal incentives when agents are motivated by status as well as or instead of money. However, the paper that is closest in methodology to our own is the paper by Hoppe, Moldovanu and Sela (2009). This paper considers signalling tournaments, and amongst other questions examines the effect on equilibrium strategies and total welfare of an increase in the dispersion of the distribution of types, using stochastic orders of a similar type to those employed here.

The paper is organized as follows. In the next section we present the model and show that strategies and equilibrium outcomes can be written in terms of rank in the distribution of income. In Section 3 we compare the two methodologies in games of status - one indexing individuals by their level of income, and the other indexing by rank. In Sections 4 and 5 we utilize the concept of dispersive ordering and conduct comparative static analysis. In Section 6 we show that our techniques can be useful for comparing individual's choices and welfare for non-rank-preserving income transformations. In the conclusion, we discuss the applicability and limitations of both methodologies.

## 2 The Model

Following Frank (1985) and Hopkins and Kornienko (2004), we consider a simple model where individuals care about their social status as determined by their relative consumption of a visible or positional good. Specifically, we assume an economy consisting of a continuum of agents, identical except in terms of income. Each individual's income  $z$  is private information and is an independent draw from a common distribution. This is described by a distribution function  $G(z)$  which is twice continuously differentiable with a strictly positive density on some interval  $[\underline{z}, \bar{z}]$  with  $\underline{z} \geq 0$ . Income is divided between expenditure on visible, conspicuous consumption and on regular non-conspicuous consumption. Agents' utility depends on status  $S$ , the absolute level of consumption

of the visible good  $x$  and the consumption of another (non-positional) good  $y$ , the consumption of which is not directly observable by other agents.

Agents' choices of conspicuous consumption are aggregated in a distribution of conspicuous consumption  $F(\cdot)$ , with  $F(x)$  being the mass of individuals with consumption less than or equal to  $x$ . Following Frank (1985) and Robson (1992), an agent's status will be determined by her position in this distribution of conspicuous consumption, with higher position meaning higher status. Following Hopkins and Kornienko (2004), we define status as follows:

$$S(x, F(\cdot)) = \gamma F(x) + (1 - \gamma)F^-(x) + S_0 \quad (1)$$

where  $x$  is individual's consumption,  $\gamma \in [0, 1)$ ,  $F(x)$  is the mass of individuals with consumption less or equal to  $x$ , and  $F^-(x) = \lim_{x' \rightarrow x^-} F(x')$  is the mass of individuals with consumption strictly less than  $x$ . The current formulation is a way of dealing with ties. For example, if all agents chose the same level of consumption in a sense they would all be "equal first", but perhaps not as happy as someone who was uniquely first. To reflect this, the current assumption would award them status equal to  $\gamma$  which is strictly less than one.<sup>2</sup> In contrast, if the distribution of consumption  $F(x)$  is continuous, there are no ties, the above measure of status is identical to rank in consumption, or  $S(x, F(\cdot)) = F(x)$ . The parameter  $S_0 \geq 0$  is a constant representing a guaranteed minimum level of status, reflecting the intensity of social pressures. We discuss its role below (following the system of equations (7)).

We follow Hopkins and Kornienko (2004), and assume that individuals have identical preferences over absolute consumptions and status as follows:

$$U(x, y, S(x, F(\cdot))) = V(x, y)S(x, F(\cdot)) \quad (2)$$

In effect,  $V(\cdot)$  is a conventional utility function over the two goods,  $x$  and  $y$ , and we assume that it is non-negative, strictly increasing in both its arguments, strictly quasiconcave and twice differentiable. We further assume that  $V_{ii} \leq 0$  for  $i = 1, 2$  and that  $V_{ij} \geq 0$  for  $i \neq j$ . This formulation assumes the convenient form that utility is multiplicatively separable in status.<sup>3</sup> Agents simultaneously decide how to allocate their endowment  $z$  between consumption of a positional good  $x$  and of a non-positional good  $y$ . Let  $p$  be the price of the positional good with the price of the non-positional good is normalized to one. Given the assumptions on  $V(\cdot)$ , the budget constraint will hold with equality and an agent with income  $z$  faces the following problem,

$$\max_x V(x, z - px) (\gamma F(x) + (1 - \gamma)F^-(x) + S_0) \quad \text{subject to } x \in [0, z/p]. \quad (3)$$

As individuals are in competition for status, their choice of consumption of different types of goods will be strategic. Note that the distribution of conspicuous consumption

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<sup>2</sup>This also gives agents an incentive to break any ties, so, as we will see, there will be no ties in equilibrium. See Hopkins and Kornienko (2004) for a full rationale of this specification.

<sup>3</sup>We believe what is important is that there is (weak) complementarity, i.e. supermodularity, between the three goods,  $x$ ,  $y$  and  $S$ . We go on to explore a more general utility formulation in Hopkins and Kornienko (2008).

$F(\cdot)$  is endogenously determined, so is social status  $S(x, F(\cdot))$ . Thus, a rational individual makes a consumption choice in anticipation of consumption choices of all other individuals, i.e. is engaged in a game of status. It is possible to solve the resulting game but the solution will, however, depend on the distribution of income in society. This game has a formal resemblance to a first-price auction, as increasing one's conspicuous consumption leads to a trade-off between the increase in status and the decrease in non-conspicuous consumption component of utility, just as a bidder in an auction trades off an increase in probability of winning for lower realized profits in the event of winning.

In games of incomplete information, it is customary to identify individuals by their type, which is a draw from an exogenous distribution of characteristics. Since in the games of status we consider, agents only differ in terms of income, one may find it natural to take an agent's type to be her income  $z$  - as it is done in Hopkins and Kornienko (2004). However, given an income distribution  $G(z)$ , an agent of income  $\tilde{z}$  has rank  $\tilde{r} = G(\tilde{z})$  and it is equally valid to think of his type as being  $\tilde{r}$  as much as it is  $\tilde{z}$ .<sup>4</sup> We can also write his income as a function of his rank or  $\tilde{z} = G^{-1}(\tilde{r})$  (i.e.  $\tilde{z}$  is at the  $\tilde{r}$ -quantile). Note an important distinction. While in everyday speech, "rank" and "status" are almost synonymous, here we use the two terms to signify two different things. Rank we take to be an agent's true, exogenous rank in the underlying distribution of income, or  $r = G(z)$ . Status  $S$ , however, is endogenously determined by visible expenditure on positional goods in comparison with the expenditures of others according to the function (1). There are several advantages to this rank-based approach which we discuss in Section 3.

Thus, in this paper we define the game of status as the game between a continuum of agents each having type  $r \in [0, 1]$ , income  $G^{-1}(r)$  and payoff function (2). All agents make a simultaneous choice of conspicuous consumption  $x$ , subject to a constraint that it is feasible given income  $G^{-1}(r)$ , or  $x \in [0, G^{-1}(r)/p]$ . Following the convention in games of incomplete information, an agent's strategy is a mapping from type to action and, thus, here a strategy is  $x(r) : [0, 1] \rightarrow [0, G^{-1}(r)/p]$ , a map from rank in income to expenditure on the conspicuous good. By the same convention, a symmetric Nash equilibrium is one in which all agents use the same strategy.<sup>5</sup> So, to summarize, the game can be written formally in terms of players indexed by their rank, the relation between their type and their income given by the inverse distribution function, action space, strategy set and payoffs:  $\{r \in [0, 1]; G^{-1}(r) : [0, 1] \rightarrow [\underline{z}, \bar{z}]; x \in [0, G^{-1}(r)/p]; x(r) : [0, 1] \rightarrow [0, G^{-1}(r)/p]; V(x, G^{-1}(r) - px)S(x, F(\cdot))\}$ .

To find such an equilibrium, suppose all agents adopt the same strictly increasing, differentiable strategy  $x(r)$  and consider whether any individual agent has an incentive to deviate.<sup>6</sup> Suppose that instead of following the strategy followed by the others, an

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<sup>4</sup>We have assumed that  $G(\cdot)$  is strictly increasing on its support so that there is a one-to-one relation between income and rank.

<sup>5</sup>We do not formally rule out asymmetric equilibria but they are not likely to exist given that this game is very similar to a standard first price auction that is known not to have asymmetric equilibria (see, e.g. Maskin and Riley, 2003).

<sup>6</sup>Hopkins and Kornienko (Proposition 1, 2004) showed that equilibrium strategies are necessarily



agent with rank  $r_i$  chooses  $x_i = x(\tilde{r})$ , that is, she consumes as though she had rank  $\tilde{r}$ . Note first that  $F(x_i) = x^{-1}(x_i) = \tilde{r}$ , resulting in  $S_i = S_0 + \tilde{r}$ , and second that her utility would be equal to

$$U = V(x(\tilde{r}), G^{-1}(r_i) - px(\tilde{r}))(S_0 + \tilde{r}).$$

We differentiate this with respect to  $\tilde{r}$ . Then, given that in a symmetric equilibrium, the agent uses the equilibrium strategy and so  $\tilde{r} = r_i$ , this gives the first order condition,

$$V(x_i, G^{-1}(r_i) - px_i) + (S_0 + r_i)x'(r_i) (V_1(x_i, G^{-1}(r_i) - px_i) - pV_2(x_i, G^{-1}(r_i) - px_i)) = 0. \quad (4)$$

This first order condition therefore defines a differential equation,<sup>7</sup>

$$x'(r) = \frac{V(x, G^{-1}(r) - px)}{pV_2(x, G^{-1}(r) - px) - V_1(x, G^{-1}(r) - px)} \frac{1}{S_0 + r} = \frac{\phi(x, G^{-1}(r))}{S_0 + r} \quad (5)$$

Two important points to recognize are that, first, this differential equation and the equilibrium strategy, which is its solution, both depend on the distribution of income through the relation  $z = G^{-1}(z)$ . Thus, changes in the distribution of income will change behavior. Second, the solution  $x(r)$  gives the inverse of the equilibrium distribution of consumption. That is, given that in equilibrium  $r = F(x)$ , we have  $x(r) = F^{-1}(r)$ .

Our next step is to specify what Frank (1985) and Hopkins and Kornienko (2004) call the “cooperative choice”, which is an optimal consumption choice  $(x^c(r), y^c(r))$  when an individual does not or cannot affect her social status. Specifically let us assume that status depends on underlying rank, but is independent of consumption, so that  $S = S_0 + r$ . Then consumers’ choices would correspond to the standard tangency condition:  $V_1(x^c, y^c)/V_2(x^c, y^c) = p$ . Given the budget constraint  $px^c(r) + y^c(r) = G^{-1}(r)$ , one can rewrite the above tangency condition as:

$$\frac{V_1(x^c(r), G^{-1}(r) - px^c(r))}{V_2(x^c(r), G^{-1}(r) - px^c(r))} = p. \quad (6)$$

Let  $x^c(r)$  be the strategy implied by the above condition. The cooperative strategy also is used in the boundary condition for the differential equation (5).

In fact, the equilibrium behavior of the poorest individual is quite different in the two different cases, where  $S_0$  minimum guaranteed status is zero, and when it is positive.

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strictly increasing and differentiable.

<sup>7</sup>As we will see, the analysis of equilibrium choices under rank-indexing approach is much simpler than under income-indexing approach. To see that, compare equation (5) to the equation 6 in Hopkins and Kornienko (2004):

$$\frac{dx(z)}{dz} = \left( \frac{g(z)}{S_0 + G(z)} \right) \left( \frac{V}{pV_2 - V_1} \right) = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z)$$

In other words, under income-indexing, the analysis of equilibrium decisions is complicated by the presence of income density  $g(z)$  in the differential equation. As we will see later on (see footnote 13), the reverse is true of the analysis of equilibrium utilities.

This is because in any symmetric equilibrium, the poorest individual will have the lowest status. The question is what is the optimal response to this. In fact, the initial conditions, or the choices of the individual with the lowest rank zero are:

$$\begin{aligned} S_0 = 0 &\Rightarrow x(0) = \frac{G^{-1}(0)}{p} \\ S_0 > 0 &\Rightarrow x(0) = x^c(0) \end{aligned} \tag{7}$$

When  $S_0$  is positive, the lowest ranked individual does not take part in social competition and spends only the cooperative amount on visible consumption. In complete contrast, when  $S_0 = 0$ , low ranked individuals are desperate and spend all their endowment in a futile attempt to get ahead.<sup>8</sup>

Given the smooth mapping  $z = G^{-1}(r)$ , one can adapt the result of Hopkins and Kornienko (2004, Proposition 1) to show the following.

**Proposition 1** *The differential equation (5) with boundary conditions (7) has a unique solution which is an essentially unique symmetric Nash equilibrium of the game of status. Equilibrium conspicuous consumption  $x(r)$  is greater than in the absence of status concerns, that is  $x(r) > x^c(r)$  on  $(0, 1]$ .*

The equilibrium is only “essentially” unique as, when  $S_0 = 0$ , other equilibrium behavior is possible for the agent with the lowest income. Specifically, given the necessary condition for equilibrium  $\lim_{z \rightarrow \underline{z}^+} x(z) = \underline{z}/p$ , the agent with income  $\underline{z}$  will always have rank zero and always have zero utility and so is indifferent between any choice of  $x$  on the range  $[0, \underline{z}/p]$ . The specific boundary condition that  $x(0) = G^{-1}(0)/p$ , she spends all her income, is not crucial to our analysis but is used for convenience.

Proposition 1 also implies that the equilibrium strategy  $x(r)$  is Pareto-dominated by the cooperative choice  $x^c(r)$ . Note that, given the status function (1) and that in equilibrium  $r = F(x)$ , status is  $S = r + S_0$ , exactly corresponding to underlying rank, just as in the cooperative case. However, equilibrium conspicuous consumption is higher at nearly every rank. Welfare is therefore higher at every rank, except the lowest, in the cooperative case. Thus, in equilibrium everyone spends too much. Further, as we will see, changes in the income distribution that force individuals to spend even more can make them worse off.

### 3 Indexing by Rank vs. Indexing by Income

Our principal interest is to explore the effect of changes in income distribution on each agent’s equilibrium behavior and well-being. This approach avoids the controversy

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<sup>8</sup>In effect,  $S_0 = 0$  represents a situation where being the lowest-status individual is deadly (“devil takes the hindmost”) which induces desperation. With no such threat, that is with  $S_0$  positive, the lowest-ranked individual has little incentive to compete in a race that in equilibrium he always loses.

surrounding interpersonal comparability of welfare. Furthermore, it would also be useful in issues of political economy to determine which agents benefit from interventions such as redistributive policies. However, a question arises - how do we “fix” a particular agent.

In this paper we index individuals by their rank  $r = G(z)$  in the income distribution  $G(z)$ , and write strategies and equilibrium utilities as functions of an agent’s rank  $r$ , i.e.  $x(r)$  and  $U(x(r), G^{-1}(r) - px(r), S_0 + r) = U(r)$  respectively. This is in contrast to Hopkins and Kornienko (2004), where individuals are indexed by their income  $z = G^{-1}(r)$ , and equilibrium consumption choices and equilibrium utility are written as functions of income  $z$ , or  $x(z)$  and  $U(x(z), z - px(z), S(z)) = U(z)$  respectively. As we will show in this section, the array of differences between the two approaches is surprisingly rich, and thus is worthy of clarification.

First, the income-indexing approach used in Hopkins and Kornienko (2004) utilized the comparative statics techniques developed in the literature on first-price sealed-bid auctions. While this approach to games of status has proved to be useful, its applicability is limited to the income transformations that do not change the support of the income distribution. For example, it cannot be used to explore a policy-relevant redistributive scheme which raises the income of all those with below average income, while taxing all those with above average income. To see this, imagine the lowest income ex ante was \$5000, but ex post was \$6000. We cannot compare how it feels to have \$5000 before and after the changes, as after the changes there is no-one with that income.

Second, the rank-indexing technique might be attractive to policy makers who are interested in what happens to those occupying a particular social position - say, the average or median individual, or the bottom or top decile. Instead, if a policy maker is interested in the effect of a policy that may leave the situation of some individuals unchanged, the income-indexing approach is more of use. Consider, for example, a case where every individual with income  $z$  has higher rank under the initial income distribution  $G_a(z)$  than under the stochastically higher income distribution  $G_p(z)$ , or  $G_a(z) > G_p(z)$  for almost all  $z$  (see the top right panel of Figure 1). Hopkins and Kornienko (2004) show that, in this case, equilibrium utility  $U(z)$  falls at almost every level of income, i.e.  $U_a(z) > U_p(z)$  for almost all incomes  $z$  (the bottom right panel). In other words, when the income distribution improves in the sense of stochastic dominance, so that some people’s incomes grow, someone whose own income is unchanged - say an individual with income  $z_0$ , - is made worse off by the improved situation of others (i.e.  $U_a(z_0) > U_p(z_0)$ ). That is, by making comparisons at constant income, one can look at those who were left behind by such a change, rather than at those who benefitted from it.

Third, these two different approaches generate results that sometimes can seem contradictory. Consider again the case of a first-order stochastic dominance change in the income distribution, and let us now compare utilities at a constant rank. Fix the individual with income  $z_0$  and rank  $r_0$  in the initial distribution  $G_a$ , and suppose she

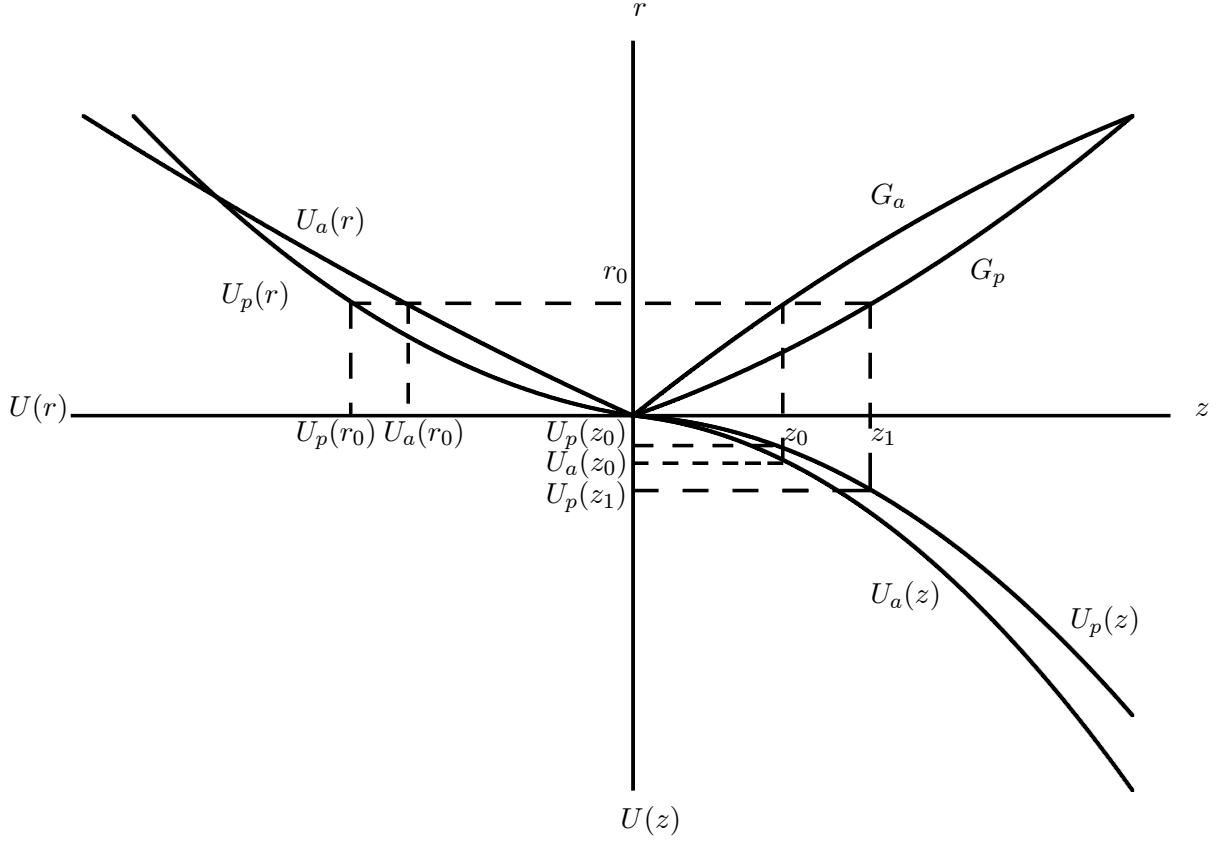


Figure 1: Comparative statics: income indexing vs. rank indexing. The top right panel represents the relationship between income and rank for two distributions  $G_a$  and  $G_p$ , with  $G_p$  being stochastically higher than  $G_a$ . The bottom right panel represents the comparative statics of welfare under the income indexing approach of Hopkins and Kornienko (2004). The top left panel represents the comparative statics of welfare under the new rank-indexing approach.

has the same rank  $r_0$  in the new distribution  $G_p$  (see the top right panel of Figure 1). Figure 1 shows a result that we will go on to prove in Section 5: this individual can have greater utility in the new distribution, i.e.  $U_p(r_0) > U_a(r_0)$  (in the top left panel). However, this is not a contradiction to the above-mentioned result of Hopkins and Kornienko (2004). This is because since she still occupies the same rank position in the new distribution  $G_p$ , she will have the higher income  $z_1$ .<sup>9</sup> Her utility of her new income  $z_1$  in the new distribution  $G_p$  exceeds her utility of her old income  $z_0$  in the old distribution  $G_a$  (the bottom right panel). Thus, the two methods are two different ways of looking at the same phenomenon.

Fourth, the two methods rely on different ways to assess social competitiveness,

<sup>9</sup>By the definition of stochastic dominance  $G_p(z) \leq G_a(z)$  for all  $z$ . This implies that, for a given income one's rank is lower in the stochastically higher distribution, and for a given rank, one's income is higher in the higher distribution.

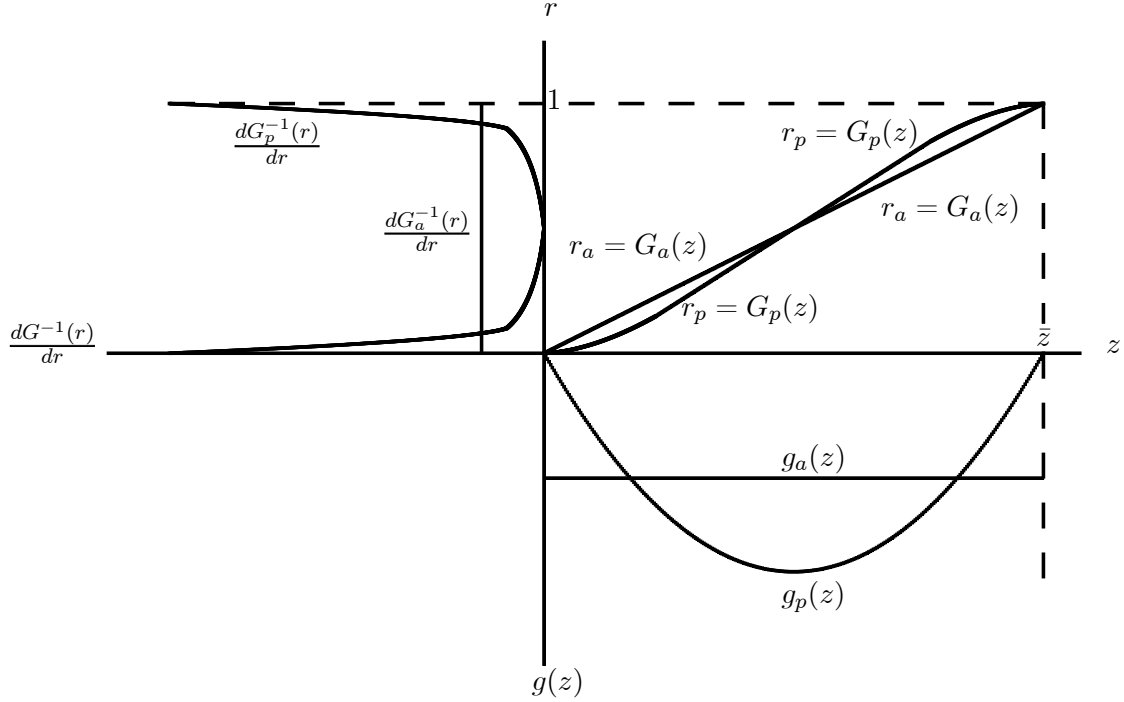


Figure 2: Densities vs. inverse densities. The top right panel represents the relationship between income and rank for two distributions  $G_a$  and  $G_p$ , with  $G_p$  being more “equal” than  $G_a$ . The bottom right panel represents the densities of the two distributions,  $g_a(z)$  and  $g_p(z)$ . The top left panel represents the inverse densities of the two distributions,  $\frac{dG_a^{-1}(r)}{dr}$  and  $\frac{dG_p^{-1}(r)}{dr}$ .

which is central to the games of status. Observe that because income and rank stand in a reciprocal relationship to each other, the density of the individual’s “rank space” is reciprocal of the density of her “income space”:

$$\frac{dG^{-1}(r)}{dr} = \frac{1}{g(G^{-1}(r))} = \frac{1}{g(z)} \quad (8)$$

Thus, the two sides of the “social coin” affect the return to happiness in a reciprocal way. In other words, take people around a given individual with rank  $r$ . The more densely packed are these individuals in the “rank space” - and, reciprocally, the more sparsely packed they are in the “income space” - the higher is the marginal return to happiness from rank, and vice versa (see Figure 2). This observation will be important for our argument to follow.

Fifth, one might be interested in total or average social welfare as well as the welfare of individuals. Under the income based approach, it was difficult to make comparisons. While average social welfare is equal to  $\int U(z)dG(z)$ , it is often difficult to calculate whether it rises. For example, in the situation illustrated in the right hand side of Figure 1, it is not clear whether average welfare is higher under income distribution  $G_a$  or distribution  $G_p$ . Individual utility is lower at each level of income under  $G_p$ . However, as  $G_p$  is stochastically higher, there are more rich people, who have high utility. Overall, the comparison is ambiguous. However, the equivalent expression for social

welfare under the rank approach is  $\int U(r)dr$ . Therefore, if utility rises at each rank, it is necessarily true that social welfare must have risen. Under the income indexing approach, it is also difficult to determine the effect of such changes on the distribution of consumption. Even if we know the solution  $x(z)$ , total consumption again depends on an integral  $\int x(z)dG(z)$  which can be difficult to interpret. However, under the rank-based approach, we have  $x(r) = F^{-1}(r)$ , so that if, for example,  $x_p(r) \geq x_a(r)$  on  $[0, 1]$ , then we know immediately that the ex post distribution of consumption stochastically dominates the ex ante distribution.

Sixth, in order to get clear-cut analytical results, each method calls for an appropriate partial ordering of income distributions. The income-indexing approach employs the concepts of first and second order stochastic dominance, as well as their strengthenings based on the likelihood ratio order, or the ratio of densities (which is the ratio of competitiveness in the income space). Here we develop novel techniques that use a less-known way of ordering distributions based on the ratio of the inverse densities (which is the ratio of competitiveness in the rank space).

In particular, here we use the dispersive ordering discussed in detail by Shaked (1982) (see also references therein) and Shaked and Shanthikumar (1994).<sup>10</sup> Let  $F$  and  $G$  be two arbitrary continuous distribution functions each with support on an interval. However, these two intervals do not have to be identical or even to overlap. Let  $F^{-1}$  and  $G^{-1}$  be the corresponding left-continuous inverses (so that  $F^{-1}(r) = \inf\{z : F(z) \geq r\}, r \in [0, 1]$  and  $G^{-1}(r) = \inf\{z : G(z) \geq r\}, r \in [0, 1]$ ), and let  $f$  and  $g$  be the respective densities. Income in the society with income distribution  $F$  is said to be smaller in the dispersive order (or less dispersed) than income in the society with income distribution  $G$  (denoted as  $F \prec_{disp} G$ ) whenever

$$F^{-1}(r_2) - F^{-1}(r_1) \leq G^{-1}(r_2) - G^{-1}(r_1) \text{ whenever } 0 < r_1 \leq r_2 < 1$$

That is, the income difference between individuals ranked  $r_1$  and  $r_2$  in both societies is no greater in the society with distribution  $F$  than in the society with distribution  $G$ . This implies that the income increase that an individual with rank  $r$  and income  $z$  gets by moving from the distribution  $F$  to more dispersed distribution  $G$  is progressive with both rank  $r$  and income  $z$ , i.e.  $G^{-1}(r) - F^{-1}(r)$  is increasing in rank  $r$  for  $r \in (0, 1)$  and  $G^{-1}(F(z)) - z$  increases in  $z$ . Finally, when both distributions have finite means, if  $F$  is less dispersed than  $G$  then  $\text{Var}_F(z) \leq \text{Var}_G(z)$  whenever  $\text{Var}_G(z) < \infty$ .

In what follows we will use heavily the location-free feature of the dispersive order, which implies that adding an arbitrary constant to everyone's income in one society ( $z \rightarrow z + c$  where  $c$  is a real number) will preserve the dispersive order. This would imply that  $F \prec_{disp} G$  if and only if  $F(z - c)$  crosses  $G(z)$  at most once and, when it does cross, it crosses from below. Shaked (1982, Remark 2.3) pointed out the following

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<sup>10</sup>Hoppe, Moldovanu and Sela (2009) look at equilibrium strategies and total welfare in assortative matching models using stochastic orderings which are closely related to the dispersive ordering - see Shaked and Shanthikumar (1994).

important consequence of the location-free features of the dispersive order:

$$F \prec_{disp} G \text{ if and only if } f(F^{-1}(r)) \geq g(G^{-1}(r)) \text{ whenever } r \in (0, 1). \quad (9)$$

That is, for a fixed rank, the more dispersed distribution is less dense than the less dispersed one when compared at the corresponding incomes. Note that because the condition (9) is expressed in terms of ranks, there is no problem in comparing distributions with different, even disjoint, supports.

The location-free features of the dispersive order imply that the dispersive ordering does not have a clear relationship with first and second order stochastic dominance, concepts that may be more familiar to economists. To see that, suppose  $G(z)$  is uniform on  $[0, 1]$  and  $F_1(z)$  is uniform on  $[0, 0.5]$ , while  $F_2(z)$  is uniform on  $[0.25, 0.75]$ . One can verify that the distributions  $F_1$  and  $F_2$  are equal in the dispersive order, but  $G$  dominates both  $F_1$  and  $F_2$  in a sense of dispersive order (i.e.  $F_1 \sim_{disp} F_2 \prec_{disp} G$ ). However,  $G$  first (and thus second) order stochastically dominates  $F_1$ , but  $F_2$  second (but not first) order stochastically dominates  $G$  (and of course  $F_2$  first (and thus second) order stochastically dominates  $F_1$ ). As will be seen, for the problems that we are concerned with here the dispersive order will be more useful than stochastic dominance.

## 4 The Effects of a Change in the Distribution of Income on Equilibrium Behavior

Suppose a society experiences a change in the distribution of income, or an *income transformation* - for example, because of economic growth, or because of redistributive taxation, or for any other reason. Let us denote the ex-ante cumulative distribution of income as  $G_a(z)$ , and the ex-post cumulative distribution of income as  $G_p(z)$ , each distribution being continuously differentiable with densities  $g_a(z)$  and  $g_p(z)$  that are strictly positive on their respective supports. In addition, we make the relatively strong assumption that the two distributions are distinct from each other except at a finite number of points (specifically there is no interval in  $[0, 1]$  where the two inverse distribution functions  $G_a^{-1}(r)$  and  $G_p^{-1}(r)$  are equal).<sup>11</sup> But note that, in contrast to Hopkins and Kornienko (2004), here we do not require the two distributions to have the same support, the two distributions can have no points in common.

We will consider how changes in the distribution of income affect conspicuous consumption and welfare. We first start with equilibrium strategies. We show that an increase in income for those with low rank will raise their expenditure on conspicuous consumption.<sup>12</sup>

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<sup>11</sup>This has the advantage that consequently the respective equilibrium strategies and utilities will be similarly distinct. Without this assumption, our results would go through but in terms of weak rather than strict inequalities.

<sup>12</sup>Strictly speaking, the inequalities of Proposition 2 and subsequent results should be qualified as

**Proposition 2** *Suppose that ex post incomes are (weakly) higher, that is  $G_p^{-1}(r) \geq G_a^{-1}(r)$ , on an interval  $[0, \hat{r}]$  where  $\hat{r}$  is the point of first crossing of  $G_p^{-1}(r)$  and  $G_a^{-1}(r)$ . Then  $x_p(0) \geq x_a(0)$  and ex post equilibrium consumption is higher almost everywhere on that interval, that is  $x_p(r) > x_a(r)$  on  $(0, \hat{r}]$ .*

Note that if the ex post distribution (first order) stochastically dominates the ex ante distribution this would imply

$$G_p^{-1}(r) \geq G_a^{-1}(r), r \in [0, 1] \Leftrightarrow G_p(z) \leq G_a(z), z \in [0, \infty). \quad (10)$$

Note that this implies that after the change in question, income is (weakly) higher at every rank in society. Combined with the previous proposition we can see that in a stochastically higher distribution of income (almost) all individuals will spend more on conspicuous consumption and lead to a stochastically higher distribution of consumption (remember  $x(r) = F^{-1}(r)$ ).

**Corollary 1** *Suppose that ex post incomes are (weakly) higher everywhere,  $G_p^{-1}(r) \geq G_a^{-1}(r)$  for all  $r \in [0, 1]$ . Then ex post conspicuous consumption is almost everywhere higher, i.e.  $x_p(0) \geq x_a(0)$  and  $x_p(r) > x_a(r)$  on  $(0, 1]$ , and thus ex post consumption is stochastically higher, i.e.  $F_p^{-1}(r) \geq F_a^{-1}(r)$ .*

Note that one can restate the above corollary as follows. If the ex-post income distribution  $G_p$  first-order stochastically dominates the ex-ante distribution  $G_a$ , then the ex-post distribution of visible consumption  $F_p$  also first-order stochastically dominates the ex-ante distribution of consumption  $F_a$ . Simply put, greater affluence for all leads to greater visible consumption by all.

Consider now an income transformation where the poorest individual has an increase in income, and the ex-post distribution is more dispersed than the ex-ante one, in the sense of the dispersion order introduced in the previous section. In fact, it is easy to see that such an income transformation results in an ex-post distribution first-order dominating the ex-ante one. Thus we have the following simple result.

**Corollary 2** *Suppose that ex post income for the lowest ranked is no lower*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \quad (11)$$

*and also that ex post incomes are more dispersed*

$$g_p(G_p^{-1}(r)) \leq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_p \succ_{disp} G_a \quad (12)$$

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only holding almost everywhere. Specifically, equality between  $x_p$  and  $x_a$  is possible at isolated points where both  $G_a^{-1}(r) = G_p^{-1}(r)$  and  $g_a(G_a^{-1}(r)) = g_p(G_p^{-1}(r))$  hold simultaneously. Of course, we could rule this out with further technical assumptions designed specifically to exclude this possibility. However, the only result where such a non-generic situation may be qualitatively important is Proposition 3 below, where we cannot rule out the possibility of such a non-generic crossing of  $x_p$  and  $x_a$  at the point of interest.



then *ex post* conspicuous consumption is higher almost everywhere, that is,  $x_p(0) \geq x_a(0)$  and  $x_p(r) > x_a(r)$  on  $(0, 1]$ .

However, as we will show in the next section, the increased affluence of those at the “bottom”, by pushing those “further up” to spend more on conspicuous goods, will tend to adversely affect the welfare of those in the “middle” - regardless of what happened to their incomes. Thus, even if everyone in the economy is richer, it does not necessary mean that everyone is happier.

Proposition 2 is also instrumental in understanding what happens if the distribution of income becomes more equal, for example, when a redistributive taxation scheme is imposed. Using similar reasoning, we now look at consumption decisions when income becomes more equal, or less dispersed in a sense of the dispersive ordering. Such a transformation implies that the distributions cross at some point  $\hat{r}$  (the dispersive order and our assumption that the two distributions are almost everywhere distinct imply that there would be at most one such point). Then everyone at the lower end of the income hierarchy up to the individual with rank  $\hat{r}$  will have a higher income and spend more on conspicuous consumption. This is over and beyond the standard effect of increased income on the demand for a normal good, because the individual with rank  $\hat{r}$  will spend more on conspicuous consumption even though her income has not changed. All those who are “below” her in the income hierarchy have become richer after the transformation, and thus are able to afford more conspicuous consumption. As the result, in order to “keep up” her social rank (as determined by the relative position in the consumption hierarchy), the individual with rank  $\hat{r}$  now has to spend more.

**Corollary 3** *Suppose that ex post income for the lowest ranked is higher*

$$G_p^{-1}(0) > G_a^{-1}(0) \tag{13}$$

*and also that incomes ex post are less dispersed*

$$g_p(G_p^{-1}(r)) \geq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \succ_{disp} G_p \tag{14}$$

*and also suppose that ex post income of the highest ranked is lower*

$$G_p^{-1}(1) < G_a^{-1}(1) \tag{15}$$

*Then, ex post conspicuous consumption is higher almost everywhere for relatively low ranks. That is,  $x_p(0) \geq x_a(0)$  and  $x_p(r) > x_a(r)$  on  $(0, \hat{r}]$  where  $\hat{r}$  is the only point of crossing of  $G_a^{-1}(r)$  and  $G_p^{-1}(r)$ .*

Here, everyone at the lower end of the income hierarchy, and possibly everyone, spends more on conspicuous consumption - including the individual with rank  $\hat{r}$  whose income does not change (the location-free properties of the dispersive order together with the smaller range of  $G_p$  ensures that there is exactly one point  $\hat{r}$ ). This result

implies, by continuity of the equilibrium strategy  $x(r)$ , that those with slightly higher ranks (and thus lower incomes) also will have to spend more on conspicuous consumption even though they have lower ex-post income. In other words, the increased affluence of those at the “bottom” of the social hierarchy forces those in the “middle” to spend more on conspicuous consumption in order to “keep up” their social “place”. This can be further demonstrated by the following example.

**Example 1** Suppose  $z_p = (1 - \tau)z_a + \tau\mu_a$ ,  $\tau \in (0, 1)$ , where  $\mu_a$  is the mean income of the ex-ante income distribution  $G_a$ . This is a mean-preserving income transformation (so that  $\mu_p = \mu_a$ ) and it is equivalent to a redistributive scheme whereby everyone is taxed at a flat rate of  $\tau$  and given a lump transfer of  $\tau\mu_a$ . Those with income that is initially lower than average (i.e. with  $z_a < \mu_a$ ) get an income subsidy, while those with above average income see their incomes taxed away. An arbitrary ex-ante distribution  $G_a$  crosses the corresponding ex-post distribution  $G_p$  from above at the mean income  $\mu_a$ , so that the individual with mean income  $\mu_a$  sees no change in neither rank nor income. However, by Proposition 2 she spends more on conspicuous consumption, i.e.  $x_p(G_p^{-1}(\mu_a)) > x_a(G_a^{-1}(\mu_a))$ . By continuity, those with slightly above average incomes will also spend more on conspicuous goods, even though their incomes are lower.

## 5 The Effects of a Change in the Distribution of Income on Equilibrium Utility

In this section, we will explore equilibrium welfare in the game of status when individuals are indexed by their rank. We begin with the following important question. Given the existence of relative concerns, what kind of income transformation would guarantee that *everyone* in the economy is better off? Recall that if we take the standard self-centered approach to the consumer choice problem, the answer is trivial - increase everyone’s income, and everyone in the economy would be better off. Yet a number of empirical studies, from Easterlin (1974) onwards, have pointed out that economic growth is not unequivocally happiness-enhancing. While this is often attributed to the existence of relative concerns, there has been little formal research into this possibility.

The basic argument is that as incomes rise, a greater proportion of wealth gets assigned to conspicuous consumption. When the value of such consumption to utility is relative to the consumption of others, if all have high absolute levels of consumption then everyone might be no happier than if all had low consumption. Cooper et al. (2001) explore this idea in a multisector growth model. As we will see, similar considerations are important here as well.

We consider equilibrium utility which is  $U(r) = V(x(r), G^{-1}(r) - px(r))(S_0 + r)$ . Using the envelope theorem, one can find that the marginal change in the equilibrium

utility from change in rank is:<sup>13</sup>

$$U'(r) = \frac{dU(r)}{dr} = \frac{\partial U(x, y, S)}{\partial r} = V_2(x, G^{-1}(r) - px)(S_0 + r) \frac{dG^{-1}(r)}{dr} \quad (16)$$

That is, the marginal return to happiness from rank depends on the density of the individual's "social space". Remember that from (8),  $dG^{-1}/dr = 1/g(z)$ .

We can use this to identify one model of relative concerns where growth can be of no benefit to happiness. Suppose simply that  $U = yS$ . That is, conspicuous consumption now has no intrinsic benefit and utility depends only on regular consumption and status.<sup>14</sup> Further assume that  $S_0 = 0$ . Then, we have equilibrium utility  $U(r) = (G^{-1}(r) - x(r))r$  and, importantly,  $U(0) = 0$ . The lowest ranked individual always has zero status and hence zero utility, independently of income. Further, we have  $U'(r) = r/g(G^{-1}(r))$ . That is, the slope of equilibrium utility depends only on the rank density. In words, in this model, the only thing that matters for happiness is the relative distribution of income in society, not the absolute level. So, combined with the unchanging boundary condition  $U(0) = 0$ , one can see that changes that do not change inequality, such as an additive increase in income for every individual, have no effect on utility. In this extreme model, if we gave an extra \$1000 to everyone, nobody would be better off.

Note there are three factors that drive this strength of this result. First, the value to conspicuous consumption in this extreme model is entirely relative (for example, it only matters to you that your car is bigger than your neighbour's not that it is higher quality than those of twenty years ago). Second, we assumed  $S_0 = 0$ , the lowest ranked individual has zero status and thus always zero utility. That is, the poorest in a society are miserable even if the society is rich. Note that if, in contrast  $S_0 > 0$  then by the boundary condition (7), the lowest ranked individual pays no attention to status and thus, like a neoclassical consumer, will always be better off with more income. Third, specifically, we did not change the shape of the income distribution. As we have seen, in these models, greater equality increases social competition.

Thus, turning to our general model, we can derive sufficient conditions for greater happiness for all by taking into account the three issues raised above. First, given the

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<sup>13</sup>As we mentioned before in footnote 7, the analysis of equilibrium utility under rank-indexing approach is more difficult than under income-indexing approach. To see that, compare equation (16) with the analogous equation under income-indexing approach (found in the proof of the Proposition 2 of Hopkins and Kornienko, 2004):

$$\frac{dU(z)}{dz} = \frac{dU(r)}{dr} \frac{dr}{dG^{-1}(r)} = \frac{dU(r)}{dr} g(z) = V_2(S_0 + G(z))$$

Thus, under rank-indexing, the analysis of equilibrium utilities is complicated by the presence of the reciprocal of the income density,  $1/g(z)$ .

<sup>14</sup>This specification does not exactly fit our assumptions in Section 2. However, it is easy to verify that a strictly increasing symmetric equilibrium exists in this case also. Second, it is possible to approximate the results of this extreme model within our framework by using a utility function such as  $x^a yS$  with  $a$  very close to zero.

value of consumption is not entirely relative, higher income can raise welfare. But, second, we must look after the individual at the bottom of the social hierarchy, so there is the Rawlsian-style requirement that she is made richer. Third, one needs a reduction in “competitive pressures” which can be ensured by a greater dispersion of income.

Again we compare outcomes under an ex ante distribution of income  $G_a$  and an ex post distribution  $G_p$ . The respective equilibrium utilities are  $U_a(r) = V(x_a(r), G_a^{-1}(r) - px_a(r))(S_0 + r)$  and  $U_p(r) = V(x_p(r), G_p^{-1}(r) - px_p(r))(S_0 + r)$ . We begin with an intermediate result, and look at an arbitrary income transformation that leaves some people with both unchanged incomes and ranks, and among these individuals, we look at the poorest one. The next proposition shows that such an individual will be worse off whenever those below her in the income hierarchy become more affluent.<sup>15</sup>

**Proposition 3** *Suppose that incomes are weakly higher ex post,  $G_p^{-1}(r) \geq G_a^{-1}(r)$ , on an interval  $(0, \hat{r})$ , where  $\hat{r}$  is the point of the first crossing of  $G_a^{-1}$  and  $G_p^{-1}$ . Then the utility of the agent at that crossing point is lower ex post,  $U_p(\hat{r}) < U_a(\hat{r})$ .*

This result says that if incomes rise but some individual at rank  $\hat{r}$  does not have her income increased, she is not indifferent but is strictly worse off. This is because the social pressure of others’ higher incomes forces her to increase conspicuous consumption. Note that, by continuity, some people with rank just less than  $\hat{r}$  will also be worse off, even though they now have higher income.

One might think that in order to avoid this problem and for everyone in the society to be better off, we simply have to increase the income of everyone. Yet, as the following example demonstrates, under relative concerns, higher income at every rank in society does not imply greater happiness for all.

**Example 2** *Suppose that  $U = xyS$ , with  $S_0 = 0.01$  and  $G_a$  is uniform on  $[0, 1]$ , and consider a linear income transformation of income  $z_p = 0.25z_a + 1.5$ , so that  $G_p$  is uniform on  $[1.5, 1.75]$ . Then  $U_p(r) > U_a(r)$  for all  $r \in (0, 0.92)$ , while  $U_p(r) < U_a(r)$  for all  $r \in (0.92, 1]$ .*

Notice that in the above example  $G_p$  first order stochastically dominates  $G_a$ , and *everyone* is (vastly) richer ex-post. Under the standard self-centered paradigm, that would imply that everyone would be happier ex-post as well. Yet this is not the case and indeed only 90 percent of the population is happier. This is because the transformation that raised incomes also compressed them. More compressed distributions give rise to greater social competition.

That is, in the games of status not only income, but also the degree of social competitiveness that matter. This gives rise to simple sufficient conditions for everyone

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<sup>15</sup>An implication of the issue we raised in footnote 12 is that it is possible that  $U_p(\hat{r}) = U_a(\hat{r})$  in the non-generic situation we identified there.

to be happier: a rise in incomes without a rise in social competition. For this to be true, the distribution of income should become no more compressed in the sense of the dispersive order.<sup>16</sup>

**Proposition 4** *Suppose that the income of the lowest individual is no lower ex post*

$$G_p^{-1}(0) \geq G_a^{-1}(0) \quad (17)$$

*and also ex post incomes are more dispersed*

$$g_p(G_p^{-1}(r)) \leq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_p \succ_{disp} G_a \quad (18)$$

*then almost everyone is better off ex post:  $U_p(0) \geq U_a(0)$  and  $U_p(r) > U_a(r)$  for all  $r \in (0, 1]$ .*

In other words, a sufficient condition for everyone to be happier, is that, first, the poorest person is no worse in terms of income (and thus of happiness - reminiscent of Rawls' criterion) and, second, there is a decrease in "competitive pressures" (as represented by the income density of people with similar rank). Note that together they imply that incomes are strictly higher at every rank. (Intuitively, since the lowest person is no worse off, the only way to make incomes more dispersed is to spread them upwards.) However, recall that we saw earlier in Example 2 that a general increase in income may not be sufficient to increase happiness if the "competitiveness criterion" of the equation (18) is not satisfied. That is, if incomes increase with an increase in social density, then the effect on utility is ambiguous. Increased incomes may or may not compensate for the increased social pressure.

We have looked at the case where the income transformation results in an increase in incomes. We now look at what happens when there is redistribution of a fixed level of total income.

**Example 3** *Suppose that  $U = xyS$ , with  $S_0 = 0.01$ , and  $G_a$  is uniform on  $[0, 1]$ , and consider a linear income transformation  $z_p = 0.5z_a + 0.25$ . Then  $U_p(r) > U_a(r)$  for all  $r \in (0, 0.36)$ , while  $U_p(r) < U_a(r)$  for all  $r \in (0.36, 1]$ .*

Notice that the income transformation of the above example is a mean-preserving redistribution scheme of the type considered in Example 1. Here, the bottom half of the population is richer ex-post, yet only the poorest 70 percent of them (and thus the poorest 35 percent of the entire population) is better off! The remaining 30 percent of those whom become richer (which is 15 percent of the population) and all of those who became poorer (that is the 50 percent of the entire population) is worse off. That is, the above mean-preserving scheme, by redistributing income from the richest half of the

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<sup>16</sup>In the extreme model, where  $U = yS$  and  $S_0 = 0$ , if incomes are strictly more dispersed (inequality (18) holds strictly) then utility will be higher on  $r \in (0, 1]$ .

population to the poorest half, makes happier the poorest 35 percent of population, at the expense of the decreased happiness of 65 percent of the population.

We can generalize the above example as follows. Suppose we change the income distribution so that it is less dispersed than before. We do this by increasing the incomes of those with low ranks and decreasing the incomes of those with high ranks, and increasing social competitiveness. Of course, this is consistent with the examples of redistributive policies above. Then, we can show that the middle and upper classes are worse off in this more equal society.

**Proposition 5** *Suppose that the income of the lowest ranked individual is higher ex post*

$$G_p^{-1}(0) > G_a^{-1}(0) \quad (19)$$

*and also ex post incomes are less dispersed*

$$g_p(G_p^{-1}(r)) \geq g_a(G_a^{-1}(r)) \text{ for all } r \in (0, 1) \Leftrightarrow G_a \succ_{disp} G_p \quad (20)$$

*and also suppose that ex post that highest ranked individual has lower income*

$$G_p^{-1}(1) < G_a^{-1}(1) \quad (21)$$

*Then, utility is weakly higher ex post for the poorest but falls for the middle and upper classes:  $U_p(0) \geq U_a(0)$  and  $U_p(r) < U_a(r)$  for all  $r \in [\hat{r}, 1]$  where  $\hat{r}$  is the only point of crossing of  $G_p^{-1}(r)$  and  $G_a^{-1}(r)$ .*

Notice that  $G_p$  is less dispersed than  $G_a$  in the sense of the dispersive order. The result implies that a simple redistributive scheme, which takes from those with above average income to supplement the income of those with below average income, will reduce the utility of the middle and upper classes. Of course, it is unsurprising that the rich would not benefit from such a scheme. However, an individual whose income is unchanged (the one with rank  $\hat{r}$ ) is strictly worse off and by continuity this will also apply to some “lower middle class” individuals who have less than average income. This is even though they have higher income post distribution. This is because the more compact income distribution after tax implies greater social competition, greater expenditure on conspicuous consumption and lower utility.

We lastly point out that as long as the social competition is not too extreme (i.e.  $S_0 > 0$ ) individuals at the very bottom of the distribution will be strictly better off when the very poorest individual is strictly richer.

**Proposition 6** *Suppose that the income of the lowest ranked is higher ex post*

$$G_p^{-1}(0) > G_a^{-1}(0) \quad (22)$$

*Then, if  $S_0 > 0$ , the utility of the low ranked is higher ex post: there exists  $\tilde{r} \in (0, 1]$  such that  $U_p(r) > U_a(r)$  on  $[0, \tilde{r})$ .*

That is, as long as the poorest individual does not face extreme social competition, additional income for the poorest will make them better off. This is because when  $S_0 > 0$ , the lowest ranked do not take part in social competition (see (7)) and so do not waste the additional income on conspicuous consumption. One should not take the result that the very poor may benefit from extra income as trivial, because, as the next example shows, once the poorest individual faces extreme social competition of  $S_0 = 0$ , those at the very bottom of the distribution can be worse off with higher income, as any benefits are dissipated by high social competition.

**Example 4** *Suppose  $U = yx^aS$ , with  $S_0 = 0$ . Then we have  $U(0) = 0$  irrespective of income and  $U'(r) = x^ar/g(r)$ . Suppose we look at an income transformation such as in Proposition 6 that raises the income of the poorest at the same time as increasing the density. The utility of the poorest is unchanged at zero, but if  $a$  is small and the change in density large,  $U'_p(r) < U'_a(r)$  in the neighbourhood of  $r = 0$  and so those at the very bottom of the distribution are worse off in the more equal situation.*

## 6 Non-Rank-Preserving Income Transformations

We have been trying to emphasize that, when relative concerns are present, changes in the income distribution can affect agents through three channels: one's own income, one's relative position or rank in the distribution, and the shape of the distribution. The methodology of Hopkins and Kornienko (2004) allows one to analyze individual decisions and utility for fixed incomes, but is limited to comparing distributions with the same support. Our current methodology allows one to make this analysis for fixed ranks, and thus is particularly suitable for analyzing rank-preserving income transformations (such as linear transformations of the form  $z_p = a + bz_a$ ).

In this section, we present some situations which would seem to defeat both methods. For example, suppose that incomes change in a manner that is not rank preserving and which also changes the support of the distributions. While we have no formal results for this case, nonetheless, one can still get some understanding of what happens employing the rank-indexing approach augmented by the use of a rank transformation function. Moreover, these situations highlight the importance of the third factor of analysis - the social competitiveness (defined as  $g(z)$  in income space or  $\frac{dG^{-1}(r)}{dr}$  in rank space).

**Example 5** *Suppose  $U = xyS$ , with  $S_0 \geq 0$ . Suppose  $G_a$  is uniform on  $[1, 2]$ , and consider an inequality-reducing transformation whereby that the “lower half” (or everyone with ex ante incomes of  $[1, 1.5]$ ) receives an income subsidy of 0.5, but the “upper half” (or everyone with ex ante incomes of  $[1.5, 2]$ ) sees no change in incomes. Then all incomes are now distributed according to  $G_p$ , which is uniform on  $[1.5, 2]$ .*

Here ex post everyone faces a uniform distribution on  $[1.5, 2]$ , and their conspicuous consumption choices are still determined by the equation (5). Given that the ex post

distribution first order dominates the ex ante distribution, by Corollary 1, for every fixed rank, everyone spends more on conspicuous consumption. However, here, the rank-indexing approach alone is not sufficient to analyze what happens to equilibrium utility of every individual. This is because this is an example of non-rank-preserving income transformation since in the new, post-transformation, society, individuals with ex-ante ranks of 0 and 0.5 both now have rank of 0, individuals with ex-ante rank of 0.1 and 0.6 both now have rank of 0.2, individuals with ex-ante rank of 0.49 and 0.99 both now have rank of 0.98, and so on.

However what we can do is to carry on the analysis based on rank indexing by constructing a rank transformation  $\mathcal{R} : r_a \rightarrow r_p$  implied by this income transformation. Here it is,  $r_p = 2r_a$  for  $r_a \in [0, 1/2)$  and  $r_p = 2r_a - 1$  for  $r_a \in [1/2, 1]$ . As the result, the ex-ante “lower half” sees an increase in both income and rank, while the “upper half” sees a decrease in rank and no change in income. Clearly, the bottom individual (i.e. the one with ex ante rank of 0) will have an ex post rank of 0 and a higher income, and thus will be no worse off after the income transformation. But those near the top of the bottom half (e.g. the one with ex ante rank just below 0.5) will have an ex post rank of just below 1 and higher income and will be better off. In contrast, everyone in the ex-ante “upper half” sees no change in incomes, but a decrease in ex-ante rank, and an increase in density of individuals around them. In fact, one can show that they will be worse off after the income transformation. Thus, using rank-indexing method, combined with rank transformation, one can do analysis for non-rank-preserving income transformations.

Non-rank-preserving transformations in fact highlight the importance of the third factor affecting one’s choices and well-being, namely, the density of the individual social space, or social competitiveness. As we pointed out before, an increase in equality implies an increase in social competitiveness. While in the above example the effect was not strong enough to offset the effect of increased income, Example 6 below demonstrates that one should not discard the importance of the social density. That is, one can experience an increase in income and increase in rank, but a decrease in utility because of an increase in social density, and thus an increase in competitive pressures.

**Example 6** Suppose  $U = xyS$ , with  $S_0 = 0.1$ . Suppose  $G_a$  is uniform on  $[1, 2]$ , and consider the following inequality-reducing transformation of income: everyone with incomes of  $[1, 1.1)$  receive an income subsidy of 0.9, everyone with incomes of  $[1.1, 1.2)$  receive an income subsidy of 0.8, and so on, with everyone with incomes of  $[1.8, 1.9)$  receiving an income subsidy of 0.1, while those with incomes of  $[1.9, 2.0]$  face no change in income. Then all incomes are now distributed with  $G_p$ , which is uniform on  $[1.9, 2]$ .

Again, this is not a rank-preserving income transformation since in the new, post-transformation, society, everyone with ex-ante incomes of 1, 1.1, 1.2,  $\dots$ , 1.9 (and thus ex-ante ranks of 0, 0.1, 0.2,  $\dots$ , 0.9) now have ex-post rank of 0, everyone with incomes of 1.05, 1.15, 1.25,  $\dots$ , 1.95 (and thus ranks of 0.05, 0.15, 0.25,  $\dots$ , 0.95) now have income rank of 0.5, everyone with incomes of 1.099, 1.199, 1.299,  $\dots$ , 1.999 (and thus ranks



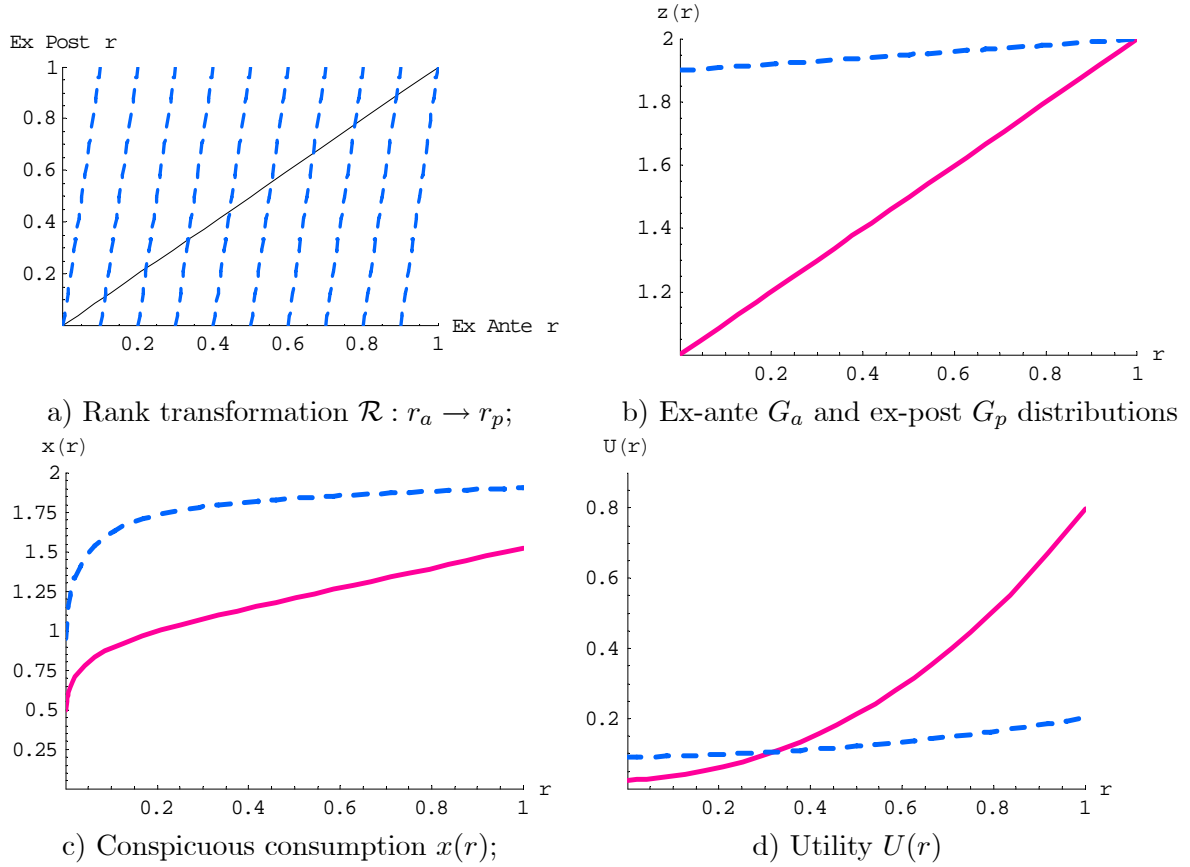


Figure 3: Social competitiveness  $g(z)$  is important. When most individuals receive an increase in income, while being “squeezed” into a small income range, the increased affluence of the “bottom” of the society pushes the rest of the society to increase their conspicuous consumption  $x(r)$  in order to “keep up”. This may result in some individuals getting lower utility even though both their income and rank increase. Ex-ante choices and utilities are represented by solid curves, while ex-post ones are represented by dashed curves. (Example 6:  $G_a$  is uniform on  $[1, 2]$ ,  $G_p$  is uniform on  $[1.9, 2]$ ,  $U = xyS$ ,  $S_0 = 0.1$  ).

of 0.099, 0.199, 0.299, ..., 0.999) now have ex-post rank of 0.99. The corresponding rank transformation  $\mathcal{R} : r_a \rightarrow r_p$  implied by this income transformation is as follows:  $r_p = 10r_a$  for  $r_a \in [0, 0.1)$ ;  $r_p = 10(r_a - 0.1)$  for  $r_a \in [0.1, 0.2)$ , ..., and  $r_p = 10(r_a - 0.9)$  for  $r_a \in [0.9, 1]$  (Figure 3a). Here, ex post everyone faces a uniform distribution on  $[1.9, 2]$  (Figure 3b), and thus, given their ex-post income and rank, their conspicuous consumption choices are determined by the equation (5) (Figure 3c).

One can see from Figure 3d that almost two thirds of the society are worse off after the transformation, even though 90 percent faced an increase in income. While the individuals varied in what happened to their rank (those above the 45 degree line in Figure 3a faced an increase in rank, those below the line faced a decrease, those on the line faced no change), yet uniformly all faced an increase in social competitiveness -

which as we argue, is important in games of status. For example, consider an individual with ex-ante income of 1.45 and rank 0.45, whose ex-ante utility is 0.17. After she receives a subsidy of 0.5, her ex-post income increases to 1.95, her rank increases to 0.5, but her utility goes down to 0.12. Needless to say, those with ex ante rank  $[0.9, 1]$  who had no change in income, now have lower rank, and face higher competitive pressures, and thus are worse off.

## 7 Conclusions

In this paper we examined games of status where a large population of individuals compete on the basis of conspicuous consumption. We find that greater equality can increase competitiveness. We find, therefore, that a sufficient condition for an increase in each individual's equilibrium utility involves non-decreased income for the society's poorest individual, plus an increase in the dispersion of incomes. Together, these two conditions ensure that income at each rank level increases (and thus an *increase* of social affluence in terms of first-order stochastic dominance), and in equilibrium, results in higher utility at each rank level.

These results involve a novel methodology where agents are indexed by their rank, rather than by income - as it was done in Hopkins and Kornienko (2004). In this type of games, the results can appear surprisingly different, even though the fundamental insights remain robust to the change in the indexing paradigm. For example, under the income-indexing approach, an increase in social affluence in terms of first-order stochastic dominance leads to a *decrease* in equilibrium utility at every level of income. Yet these apparently contrasting results are nonetheless consistent. The income-indexing approach looks at individuals whose incomes were left unchanged by an income transformation. Thus, those who did not gain from an increase in social affluence, experience a fall in utility because of the greater social competition from those who did. In contrast, when one compares ex ante and ex post utilities at a constant rank, under this type of transformation an individual of a given rank will have a higher income. If the benefits of higher income outweigh greater social competition, utility will rise. But this does not conflict with the result that individuals whose income is unchanged would see a fall in utility.

Another important insight is that, regardless whether we index individuals by their rank or their income, those whose income is left unchanged by the income transformation, tend to be adversely affected by the increased affluence of those who are below them in the income hierarchy. This is because, in order to “keep up” their social position, they have to increase their consumption of conspicuous goods further above what would be privately optimal, leading to a decline in welfare. That is, it is the unchanged income of the poor that was the main culprit behind the “equality hurts the poor” result in Hopkins and Kornienko (2004). Once we allow the income of the poor to increase (which sometimes may lead to greater equality), the poor will be better off.

Our third main insight is that in the games of status not only income and rank, but also social density, or competitiveness, matter. We demonstrate that one can have higher income and higher rank, but nevertheless have lower well-being because of the increase in social competitiveness. Note that such income transformation is non-rank preserving, yet we still were able to utilize the rank-indexing approach by augmenting it with a rank transformation function. In fact, income indexing approach can be reduced to a rank indexing approach with a suitable rank transformation.

Thus, given the similarities and differences in the two approaches, the question arises - if one is interested in analyzing what happens when relative concerns are present, which approach should one take? We can think of at least two situations where the income indexing approach might be well suited. The first such case is where income transformations differentially affect different types of income - for instance, it affects only unearned income but does not affect earned income, and vice versa, without changing the lowest income. Given that individuals may vary in their income composition, income-indexing can help to understand what happens to individuals who have only one type of income (say, earned). Another situation would be one where individuals make decisions based on their unchangeable *abilities*, or talents, and suppose that immigration changed the composition of abilities in the country. Here, the endowment-based approach would allow one to compare the decisions and well-being of domestic workers whose abilities were left unchanged by the influx of new people.

However, we now think that the rank-indexing approach is likely to be more generally fruitful in problems of social comparison.<sup>17</sup> This is because it happens to be easier, allows for a wider set of comparative static predictions, and seems to make more intuitive sense. We also hope that this theoretical investigation will prove useful for empirical analysis of the links between the distribution of income and happiness. In particular, it makes clear that the results one obtains will depend on one's choice of the method of social comparison.

Nonetheless, both our earlier and present papers show that, when relative concerns are present, people will respond to an increase in the affluence of those who are below them by increasing their own conspicuous consumption in order to maintain their social position. Thus, whether we take the rank- or income-indexing approach, any policy that increases incomes of the lower classes without a sufficient increase in the incomes of the middle classes, will tend to decrease the latter's welfare. Depending on the extent of the policy, it may make the upper classes worse off as well. In other words, whichever method of comparison is used, in these models of relative concerns, the middle classes may be worse off with policies that change the distribution of endowments in a way that has been traditionally considered to be progressive.

We should point out, however, that this work is best interpreted as an investigation of the logical consistency of the argument that social preferences such as desires for rank or status imply that greater equality is necessarily beneficial. While we find that in fact

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<sup>17</sup>We hope that the techniques developed here could also be used to advantage in first price auctions.

greater equality in this type of model increases social pressures to consume, this is not in itself an argument for maintaining or extending existing inequality. For example, we do not find that greater inequality makes everyone better off. In contrast, it is well known that when people have status concerns, appropriate income or consumption taxes can lead to Pareto improvements. The contribution of the more recent literature, such as Corneo (2002) and Hopkins and Kornienko (2004), is to show that the structure of the appropriate tax depends heavily on the underlying distribution of income.

Finally, we would like to highlight that the theoretical work on relative concerns is still in its early stages, and thus does not take into account the full diversity of real world social interactions. Here, we consider the importance of relative concerns for consumer behavior, but there is significant evidence that such concerns are also important in labor markets (see, for example, Brown et al., 2008). Furthermore, in this paper, we look at individuals that are symmetric in a sense that they are identical in all aspects apart from their endowments. Future research will, undoubtedly, look into various forms of asymmetry among individual agents. One form of asymmetry involves the possibility that, rather than comparing herself to the entire population, each individual may compare herself instead to some - possibly idiosyncratic - subset of the population. Another would be variation in the concern with status, and, moreover, this degree could depend on one's social rank. For example, Hopkins and Kornienko (2008) allow for the return to status, the rewards in society, to vary in their dispersion and find that this inequality of rewards has almost the opposite effect to the inequality of income considered here.

## Appendix

**Lemma 1** *Consider a pair of distributions  $G_a(z)$  and  $G_p(z)$  which are distinct from each other except at a finite number of points, both continuously differentiable with positive densities on the respective supports. If the corresponding equilibrium strategies are  $x_a(r)$  and  $x_p(r)$ , then at any point  $\check{r}$  such that  $x_a(\check{r}) = x_p(\check{r})$ , the sign of  $x'_a(\check{r}) - x'_p(\check{r})$  is equal to the sign of  $G_a^{-1}(\check{r}) - G_p^{-1}(\check{r})$ .*

**Proof of Lemma 1:** First note that, given the equation (5), we have that

$$\frac{x'_a(r)}{x'_p(r)} = \frac{\phi(x_a, G_a^{-1}(r))}{\phi(x_p, G_p^{-1}(r))} \quad (23)$$

so that any point where  $x_a = x_p$  the relative slope only depends on  $G_a^{-1}$  and  $G_p^{-1}$ , and thus the slopes are equal whenever  $G_a^{-1}$  and  $G_p^{-1}$  are equal. Furthermore, given our assumptions that  $V_{ii} \leq 0$  and  $V_{ij} \geq 0$ , we have that

$$\frac{\partial \phi}{\partial G^{-1}(r)} = \frac{V_2}{pV_2 - V_1} - \frac{V(pV_{22} - V_{12})}{(pV_2 - V_1)^2} > 0 \quad (24)$$

Thus, at any point where  $x_a(r) = x_p(r)$  we have that  $x'_a > x'_p$  (so that  $x_a$  is steeper than  $x_p$  and thus crosses  $x_p$  from below) whenever  $G_a^{-1}(r) > G_p^{-1}(r)$  (i.e. whenever ex-ante income exceeds ex-post income), and vice versa. ■

**Proof of Proposition 2:** By the boundary conditions (7), the condition  $G_a^{-1}(0) \leq G_p^{-1}(0)$  implies that  $x_p(0) \geq x_a(0)$  (i.e. that the poorest individual, now that she has more income, spends more on conspicuous consumption). Given our assumption that  $G_a$  and  $G_p$  are distinct it follows that  $G_p^{-1}(r) > G_a^{-1}(r)$  almost everywhere on  $(0, \hat{r}]$ . Thus, by Lemma 1,  $x_p(r)$  can only cross  $x_a(r)$  from below except perhaps at the finite number of points where  $G_p^{-1}(r) = G_a^{-1}(r)$ .

We first rule out that there is an interval where  $x_p(r) \leq x_a(r)$ . Suppose on the contrary there exist at least one interval  $[r_1, r_2] \subseteq [0, \hat{r}]$  such that  $x_p(r) \leq x_a(r)$ . By the continuity of  $x_a$  and  $x_p$ , it must be that  $x_p(r_1) = x_a(r_1)$ . Note that

$$\frac{\partial \phi}{\partial x} = \frac{-(V_1 - pV_2)^2 - V(pV_{21} - p^2V_{22} - V_{11} + pV_{12})}{(pV_2 - V_1)^2} < 0. \quad (25)$$

By a combination of Lemma 1 and (25), it would follow that  $x'_a(r) < x'_p(r)$  almost everywhere on  $[r_1, r_2]$ , which combined with  $x_a(r_1) = x_p(r_1)$  is a contradiction to  $x_p(r) \leq x_a(r)$  on the interval. Thus,  $x_p(r) > x_a(r)$  almost everywhere on  $[0, \hat{r}]$ .

We next rule out that  $x_p(r) = x_a(r)$  at individual points. By Lemma 1 and the previous argument that excludes intervals where  $x_p(r) \leq x_a(r)$ , this is only possible at the isolated points where  $G_p^{-1}(r) = G_a^{-1}(r)$ . But at any such point  $\tilde{r}$  on  $(0, \hat{r}]$ , as  $G_p^{-1}(r) > G_a^{-1}(r)$  almost everywhere, we have that  $g_p(G_p^{-1}(\tilde{r})) \geq g_a(G_a^{-1}(\tilde{r}))$ . Now, note that  $G_p^{-1}(\tilde{r}) = G_a^{-1}(\tilde{r}) = \tilde{z}$ . Next, we invoke the income indexing approach and consider solutions to the game in terms of income  $z$ , that is, solutions to the differential equation

$$\frac{dx(z)}{dz} = \left( \frac{g(z)}{S_0 + G(z)} \right) \left( \frac{V}{pV_2 - V_1} \right) = \frac{dx(r)}{dr} \frac{dr}{dz} = \frac{dx(r)}{dr} g(z) \quad (26)$$

that we write  $x_p(z)$  and  $x_a(z)$  for the respective distributions. Then if  $x_p(\tilde{r}) = x_a(\tilde{r})$ , it must be that  $x_p(\tilde{z}) = x_a(\tilde{z})$ . As  $x_p(r) > x_a(r)$  for  $r$  in  $(\tilde{r} - \epsilon, \tilde{r})$  for some  $\epsilon > 0$ , we must have  $x_p(z) > x_a(z)$  for incomes slightly less than  $\tilde{z}$ . Note that by Lemma 1,  $x'_p(\tilde{r}) = x'_a(\tilde{r})$ , and for the case of  $g_p(\tilde{z}) > g_a(\tilde{z})$ , it must be that  $x'_p(\tilde{z}) > x'_a(\tilde{z})$  so that  $x_p(z)$  crosses  $x_a(z)$  from below, which is a contradiction. This leaves us with the possibility, as was mentioned in footnote 12, that it is possible that  $x_p(r) = x_a(r)$  in a non-generic case of  $g_p(G_p^{-1}(\tilde{r})) = g_a(G_a^{-1}(\tilde{r}))$ . ■

**Proof of Proposition 3:** Notice that since  $\hat{r}$  is the point of crossing of  $G_a^{-1}$  and  $G_p^{-1}$ , this implies that  $G_p^{-1}(\hat{r}) = G_a^{-1}(\hat{r}) = \hat{z}$ : an agent of this rank has the same income  $\hat{z}$  ex ante and ex post. Note that as  $G_p^{-1}$  crosses  $G_a^{-1}$  from above, we have  $g_p(G_p^{-1}(\hat{r})) \geq g_a(G_a^{-1}(\hat{r}))$ . Again, for the non-generic case of  $g_p(G_p^{-1}(\hat{r})) = g_a(G_a^{-1}(\hat{r}))$ , we cannot rule out that  $x_p(\hat{r}) = x_a(\hat{r})$  and thus that  $U_p(\hat{r}) = U_a(\hat{r})$ . Yet for the case of  $g_p(G_p^{-1}(\hat{r})) > g_a(G_a^{-1}(\hat{r}))$ , Proposition 2 implies that  $x_p(\hat{r}) > x_a(\hat{r})$ , so this agent now spends more on conspicuous consumption. By Proposition 1, in equilibrium

$x > x_c$ , consumption exceeds the level that maximizes regular utility  $V$ . By the strict quasiconcavity of  $V$ , given a fixed level of income,  $V(x, z - x)$  is strictly decreasing in  $x$  for  $x > x_c$ . This implies that  $U_p(\hat{r}) > U_a(\hat{r})$ . ■

**Proof of Proposition 4:** First, as income is (weakly) higher at  $r = 0$  by the boundary conditions (7),  $x_p(0) \geq x_a(0)$  and  $y_p(0) \geq y_a(0)$ , so that  $U_p(0) \geq U_a(0)$  (i.e. as the poorest individual has no reduction in income she will not be worse off). Second, notice that  $U_p(r) > U_a(r)$  if and only if  $V_p(r) = V(x_p(r), y_p(r)) > V(x_a(r), y_a(r)) = V_a(r)$ . Finally, the condition (18) implies that

$$\frac{1}{g_p(G_p^{-1}(r))} = \frac{dG_p^{-1}(r)}{dr} \geq \frac{dG_a^{-1}(r)}{dr} = \frac{1}{g_a(G_a^{-1}(r))} \text{ for all } r \in (0, 1)$$

In other words,  $G_p^{-1}(r)$  is (weakly) steeper than  $G_a^{-1}(r)$  on  $[0, 1]$ , so that clearly  $G_p^{-1}(r) \geq G_a^{-1}(r)$  for  $r \in [0, 1]$ . Suppose that  $U_p(0) > U_a(0)$ , and suppose, in contradiction to the claim we are trying to prove, that  $U_p(r)$  equals  $U_a(r)$  at least once on  $(0, 1]$ . Denote the first such point as  $r_1 \in (0, 1)$  and notice that it must be that  $V(x_p(r_1), y_p(r_1)) = V(x_a(r_1), y_a(r_1))$ . But since by Corollary 1,  $x_p(r) > x_a(r)$  on  $(0, 1]$ , it must be that  $y_p(r) < y_a(r)$  in the neighborhood of  $r_1$ . Furthermore,  $dV_2 = V_{21}dx + V_{22}dy$ , and, given our original assumptions on  $V$ , it thus must be that  $V_2(x_p(r), y_p(r)) > V_2(x_a(r), y_a(r))$  in a neighborhood of  $r_1$ . Using the marginal utility condition (16), combined with the density condition (18), it must be that  $U'_p(r) > U'_a(r)$  in a neighborhood of  $r_1$ , so that  $U_p(r)$  can only be steeper than  $U_a(r)$ , and thus can only cross from below. Given  $U_p(0) > U_a(0)$ , we are done.

If instead we have that  $U_p(0) = U_a(0)$ , then, by the above argument which rules out that  $U_p$  can cross  $U_a$  from above, the claim can only fail if there is an interval  $(0, \tilde{r})$  on which  $U_p(r) \leq U_a(r)$ . Then, there must exist a point  $r_2 \in (0, \tilde{r})$  such that  $U'_p(r_2) \leq U'_a(r_2)$  and  $V(x_p(r_2), y_p(r_2)) \leq V(x_a(r_2), y_a(r_2))$ . But given (16) and the density condition (18), if  $U'_p(r_2) \leq U'_a(r_2)$  then  $V_2(x_p(r_2), y_p(r_2)) \leq V_2(x_a(r_2), y_a(r_2))$  at  $r_2$ , which can only happen if  $y_p(r_2) \geq y_a(r_2)$ . But this, combined with the fact that  $x_p(r_2) > x_a(r_2)$  (by Proposition 2) implies that  $V(x_p(r_2), y_p(r_2)) > V(x_a(r_2), y_a(r_2))$ , which is a contradiction. ■

**Proof of Proposition 5:** Notice again that  $U_a(r) > U_p(r)$  if and only if  $V_a(r) = V(x_a(r), y_a(r)) > V(x_p(r), y_p(r)) = V_p(r)$ . From Proposition 2, we have  $x_p(\hat{r}) > x_a(\hat{r})$ . We can then consider two cases. First, suppose that  $x_p(r) \geq x_a(r)$  on  $[\hat{r}, 1]$ . Then, as wealth for individuals with rank  $(\hat{r}, 1]$  is strictly lower ex-post than ex-ante, we have necessarily  $y_p(r) < y_a(r)$  on  $[\hat{r}, 1]$ . Now, as  $x_p(r) \geq x_a(r)$  and  $y_p(r) < y_a(r)$ , we then for some  $\tilde{r}$  can find a pair  $(\tilde{x}, \tilde{y})$  such that  $p\tilde{x} + \tilde{y} = px_p + y_p$  (that is,  $(\tilde{x}, \tilde{y})$  are feasible given ex-post wealth) but  $x_p^c < \tilde{x} < x_p$  and  $\tilde{y} = y_a$ . But then,  $V(x_p(r), y_p(r)) < V(\tilde{x}, \tilde{y}) < V(x_a(r), y_a(r))$ , and the result follows.

Suppose now instead that  $x_p(r) < x_a(r)$  for some  $r$  in  $(r_1, r_2)$  with  $r_1 > \hat{r}$ . If  $y_p(r) \leq y_a(r)$  on that interval, it is clear that  $V_p(r) < V_a(r)$  and we are done. Suppose instead that  $y_p(r) > y_a(r)$  on some interval  $(r_3, r_4)$  with  $r_4 \leq r_2$  (as incomes are lower ex post for  $r > \hat{r}$ , it must be that  $r_3 > r_1$ ). We want to rule out the possibility of  $U_p(r) \geq$

$U_a(r)$  somewhere on this interval. Now, it must be the case that  $V_p(r_3) < V_a(r_3)$  as  $x_p(r_3) < x_a(r_3)$  and  $y_p(r_3) = y_a(r_3)$ . We have  $g_p(r) \geq g_a(r)$  everywhere. Furthermore,  $dV_2 = V_{21}dx + V_{22}dy$ . Given that  $x$  decreases and  $y$  increases ex post on  $(r_3, r_4)$  and our original assumptions on  $V$ , it can be calculated that, given (16), that  $U'_p(r) < U'_a(r)$  on this interval. Combined with  $U_p(r_3) < U_a(r_3)$ , the result follows. ■

**Proof of Proposition 6:** If  $S_0 > 0$  we must have  $U_p(0) > U_a(0)$ , simply because by assumption  $G_p^{-1}(0) > G_a^{-1}(0)$ , the lowest ranked individual has strictly higher income ex post. Since by the boundary condition (7), the lowest ranked individual spends the cooperative amount and behaves like a neoclassical consumer, a strictly higher income must make her strictly better off. Higher utility on some interval  $[0, \tilde{r})$  then follows by continuity of  $U(r)$ . ■

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